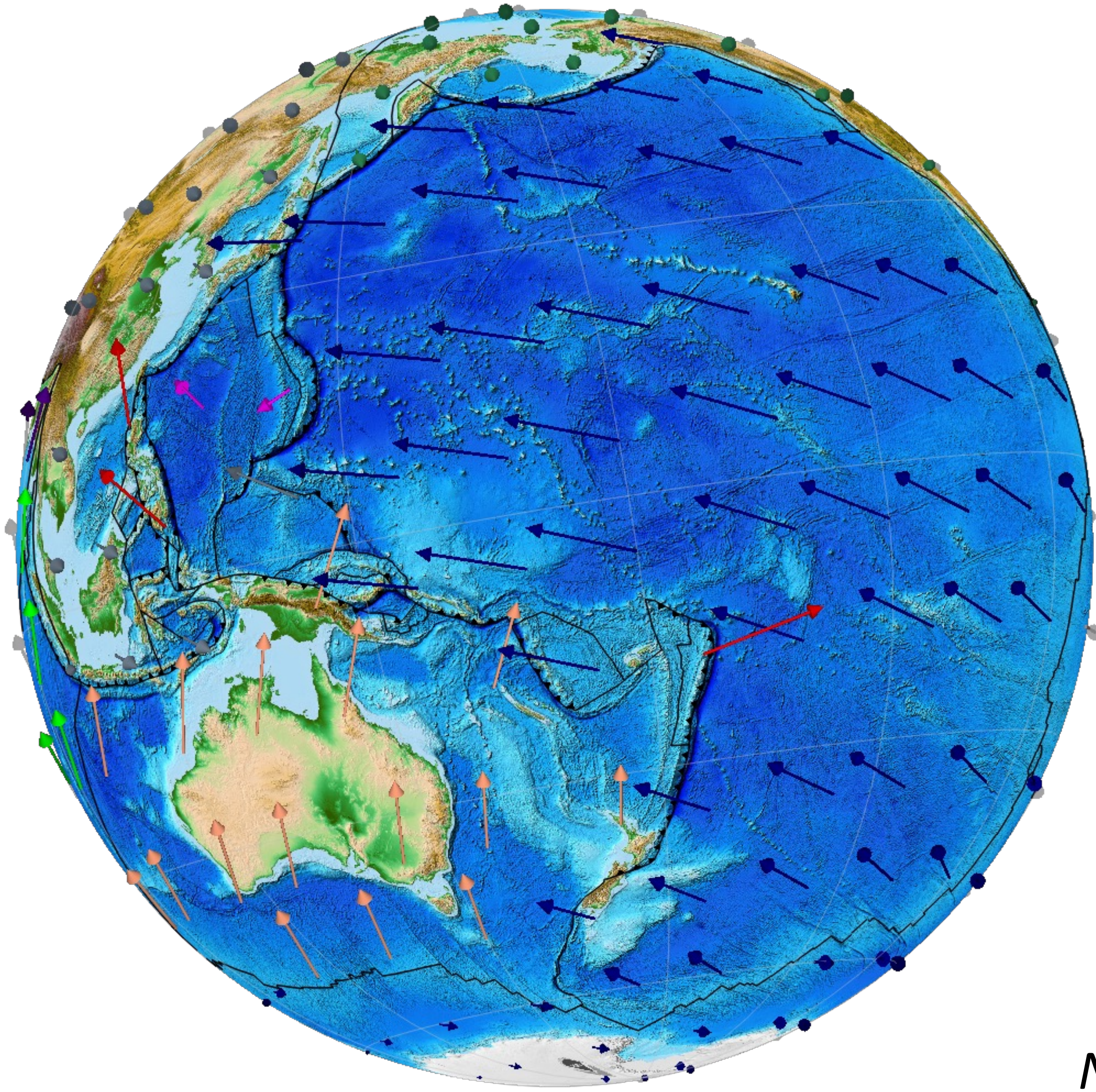
Plate tectonics, great earthquakes, and rheology across timescales

Michael Gurnis[1], Max Heldman[2],
Jiaqi Fang[1], Johann Rudi[2], Jiashun Hu[3],
Georg Stadler[4], and Nadia Lapusta[1]

1. Seismological Laboratory, California Institute of Technology
2. Dept. of Mathematics, Virginia Tech
3. Dept of Earth & Space Sciences, SUSTEC
4. Courant Institute of Mathematical Sciences, NYU

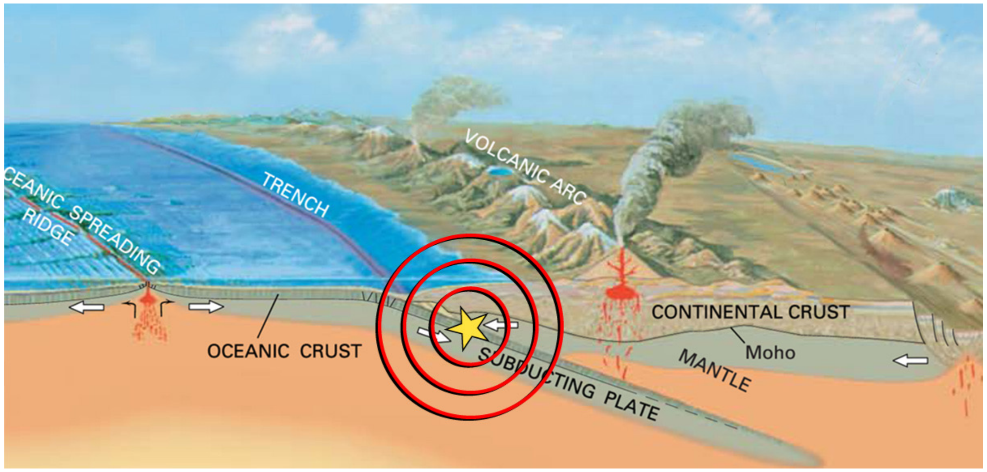
Frontera Users Meeting, Aug 5-6, 2024

Forces Driving and Resisting of Global Plate Motions



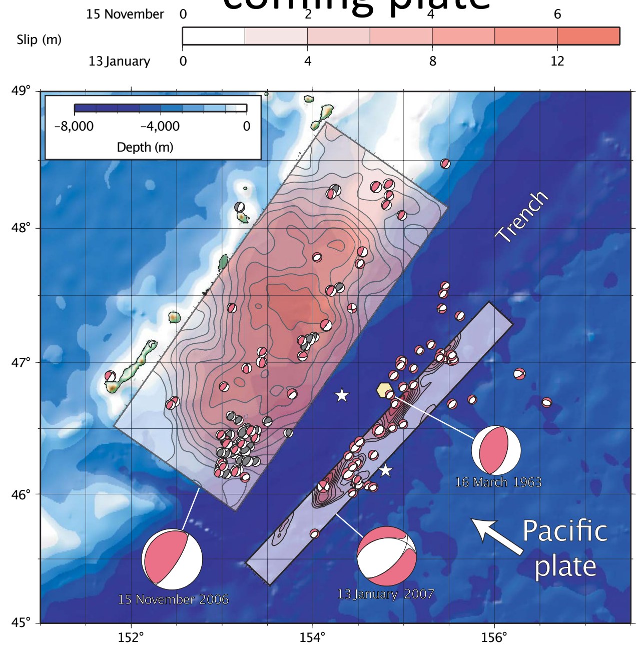
NNR, GPlates

Great Earthquakes occur on the Megathrust between plates



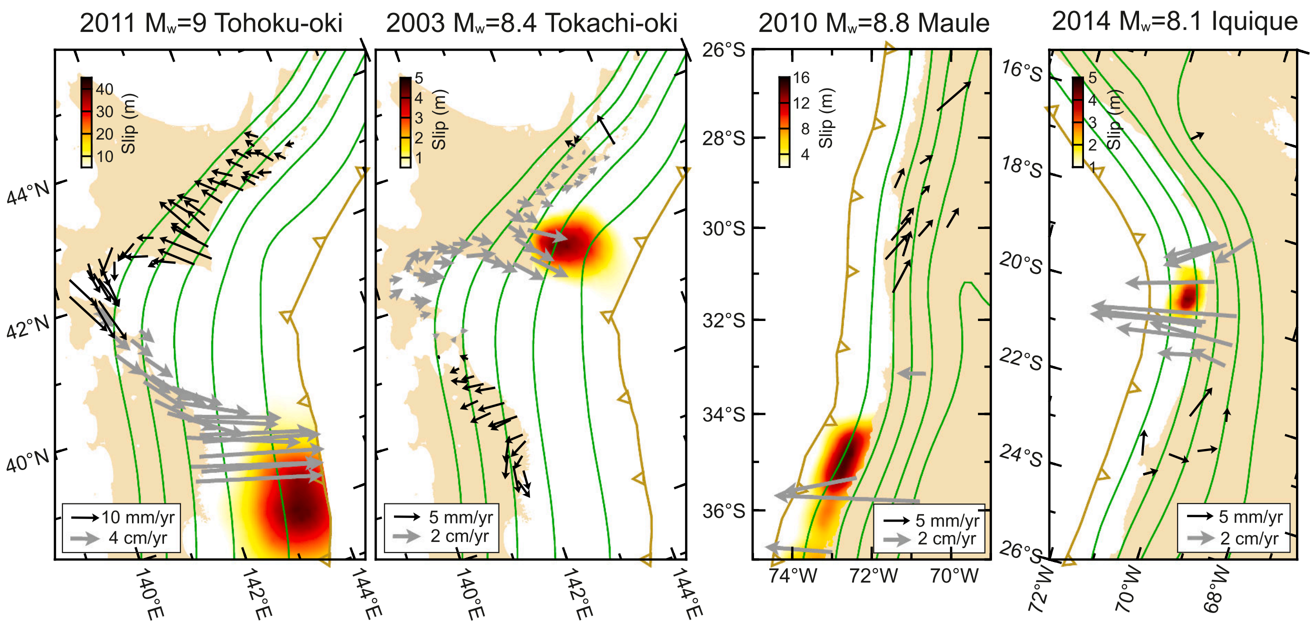
[from SAGE/EarthScope]

Stress cycling between megathrust & in-coming plate



[Ammon et al., 2008]

GNSS velocities: Post-seismic deformations (years) following great earthquakes



[Sun et al., 2024]
[Yuzariyadi and Heki , 2021]

We want to better understand some of the why's behind these & other observations

- What drives plate motions and how does slab pull work?
- What is the mechanical coupling between subduction zones based on plate motions?
- What are the trade-offs with the underlying factors which govern resistance to plate motions ?
- Is it possible to bridge the time span between seismogenic processes, megathrusts, and plate tectonics?

Forward Solution of Either Viscous Stokes or a Maxwell Material

$$\nabla \cdot u = 0$$

$$\nabla \cdot \sigma = f_b - f_e$$

$$n \times (n \times \sigma n) = 0$$

$$\sigma = pI + 2\eta\dot{\varepsilon}$$

$$\dot{\epsilon}_e + \dot{\varepsilon}_v = \frac{1}{2G}\dot{\sigma} + \frac{1}{2\eta}\sigma$$

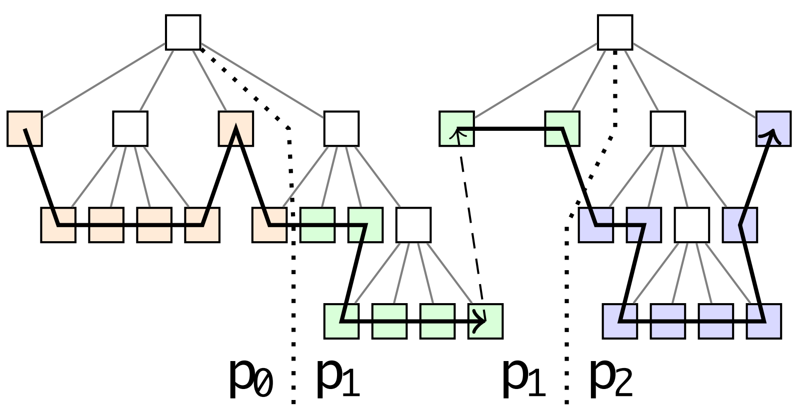
f_b and f_e are the buoyancy and unrelaxed elastic stresses

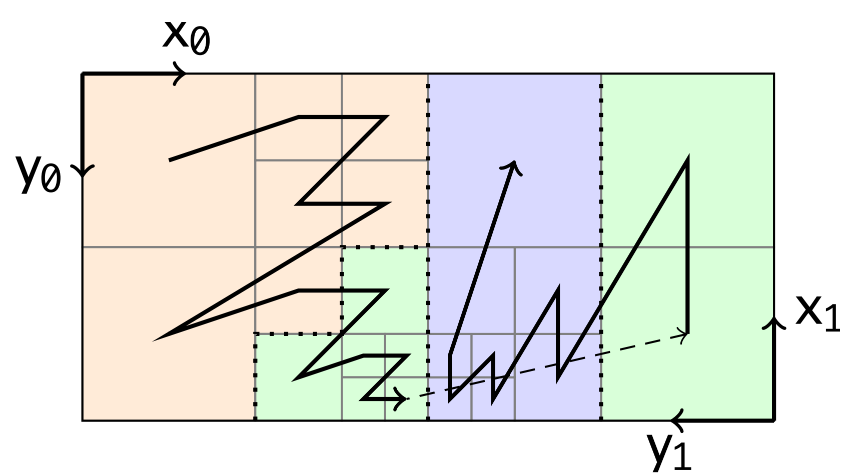
$$\eta \approx C\frac{1}{\sigma^2}\exp\left(\frac{E_a + pV_a}{RT}\right)$$

$$\mu = \mu_d + \frac{\mu_s + \mu_d}{1 + \frac{V}{V_{ref}}}$$

Rhea

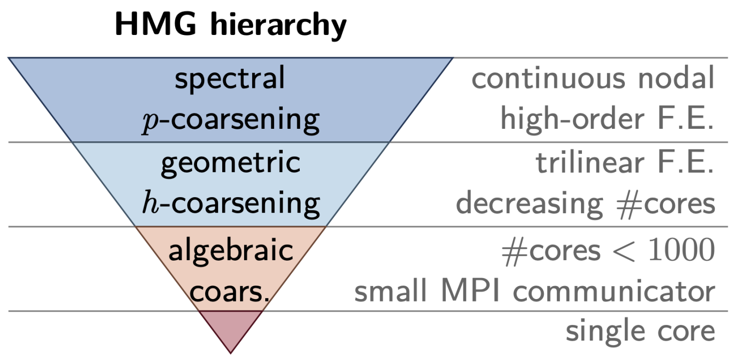
- Weak formulation of incompressible Stokes system
- Discretization with adaptive finite hexahedral elements
- Order 2 or 3 for velocity, and corresponding stable discontinuous pressure elements of lower order
- AMR (p4est) Nonlinearity is treated with Newton's method; plastic rheology uses Newton modification method to improve convergence
- Linearized Stokes solved with BFBT Schur complement, and geometric+algebraic multigrid-preconditioned GMRES
- Scales to millions of processors





[Burstedde et al. 2010, 2011; Rudi et al., 2015, 2017]

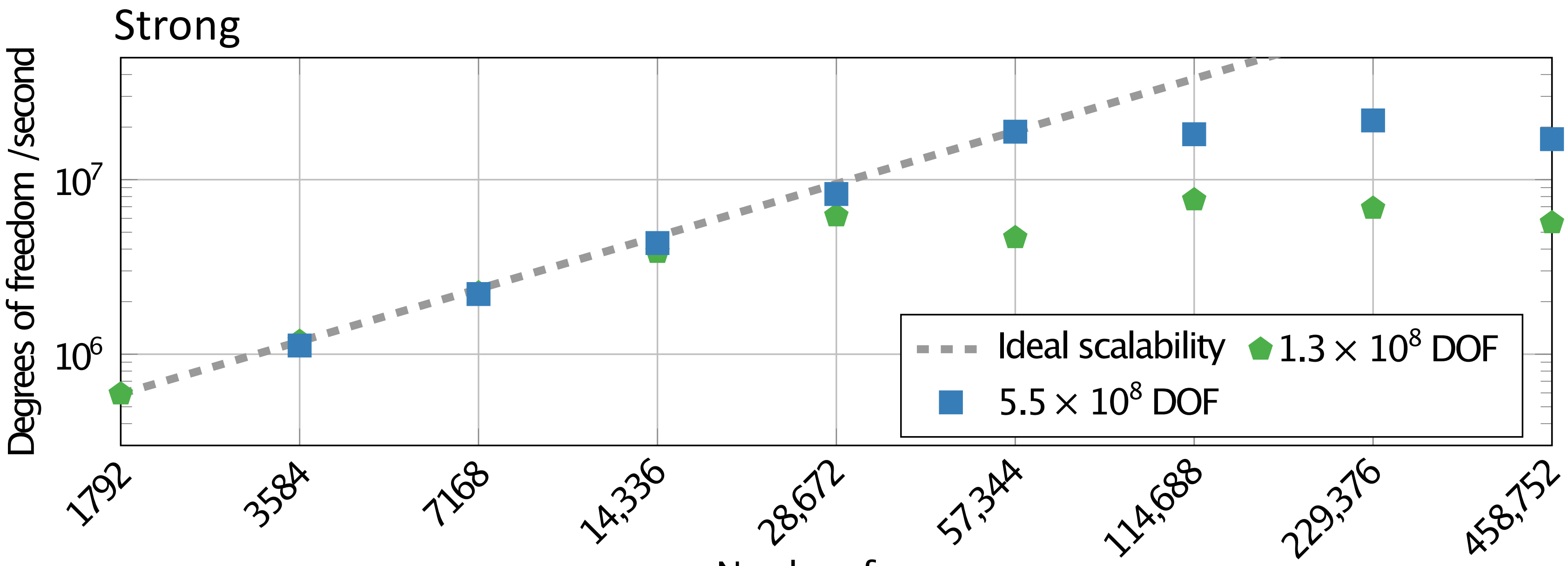
HMG: Hybrid spectral–geometric–algebraic multigrid

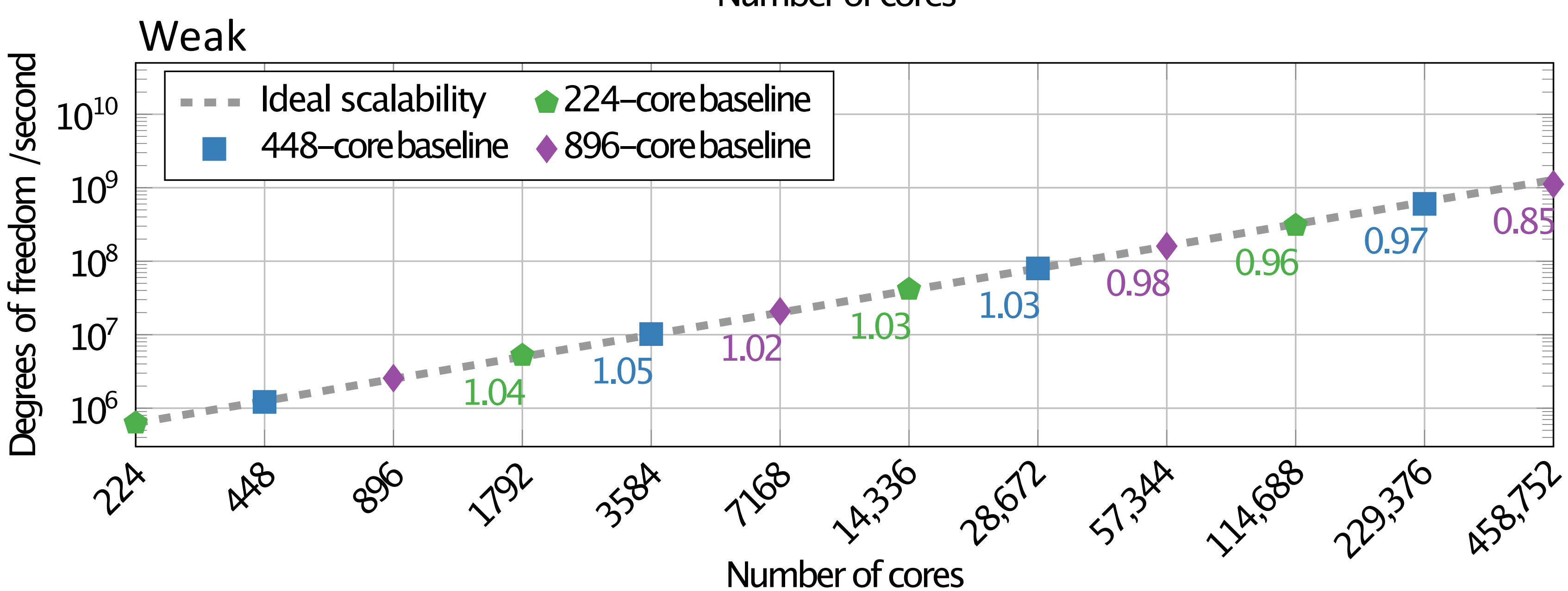


- *Multigrid hierarchy of nested meshes is generated from an adaptively refined octree-based mesh via spectral–geometric coarsening*
- *Re-discretization of PDEs at coarser levels*
- *Parallel repartitioning of coarser meshes for load-balancing (crucial for AMR); sufficiently coarse meshes occupy only subsets of cores*
- *Coarse grid solver: AMG (from PETSc) invoked on small core counts*

[Rudi et al., 2015, 2017]

Rhea Scaling on Frontera





TexaScale: Scaling on Frontera, April 2024

Science 1: Adjoint inversion: A PDE (nonlinear Stokes flow equations) constrained optimization

$$\nabla \cdot \boldsymbol{u} = 0 \qquad x \in \Omega$$

$$\nabla \cdot (\boldsymbol{\sigma_u}) = RaT\boldsymbol{e}_r \qquad x \in \Omega$$

$$\boldsymbol{n}\times(\boldsymbol{n}\times\boldsymbol{\sigma_u}\boldsymbol{n}) = 0 \qquad x \in \Gamma$$

$$\boldsymbol{\sigma_u} = -p\boldsymbol{I} + 2\eta\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{u}}$$

$$\nabla \cdot \boldsymbol{v} = 0 \qquad x \in \Omega$$

$$\nabla \cdot (\boldsymbol{\sigma_v}) = 0 \qquad x \in \Omega$$

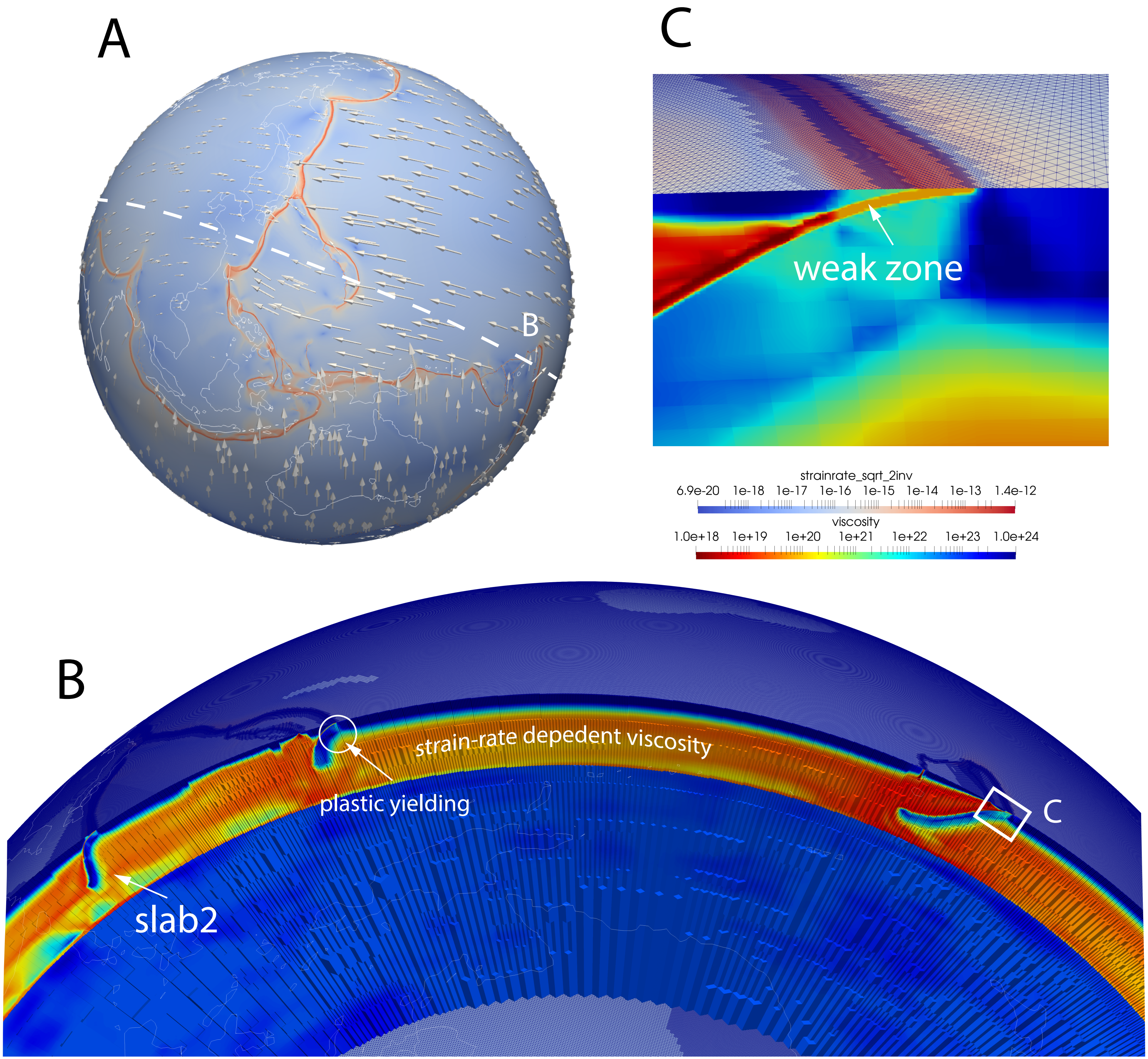
$$\boldsymbol{n}\times(\boldsymbol{n}\times\boldsymbol{\sigma_v}\boldsymbol{n}) = \boldsymbol{u}(\boldsymbol{m}) - \boldsymbol{u}_{obs} \qquad x \in \Gamma$$

$$\boldsymbol{\sigma_v} = -q\boldsymbol{I} + 2\eta\left(1 + \frac{1-n}{n}\frac{\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{u}} \otimes \dot{\boldsymbol{\varepsilon}}_{\boldsymbol{u}}}{\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{u}} : \dot{\boldsymbol{\varepsilon}}_{\boldsymbol{u}}}\right)\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{v}}$$

where $\boldsymbol{v}$, q, $\boldsymbol{\sigma_v}$, and $\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{v}}$ are ajoint velocity, pressure, stress, and strain rate.

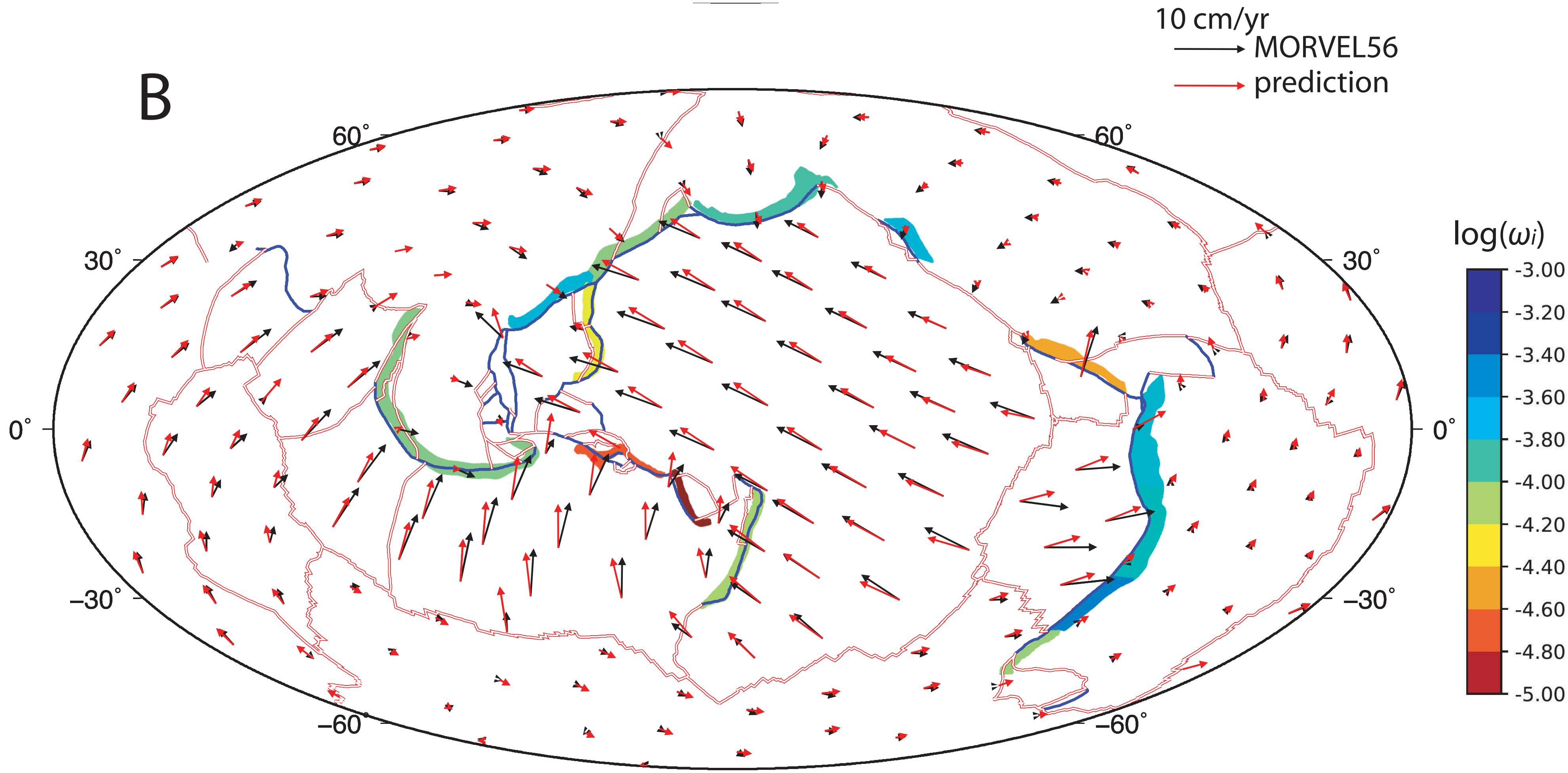
Newton's method, compute the second derivative of the objective function (Hessian)

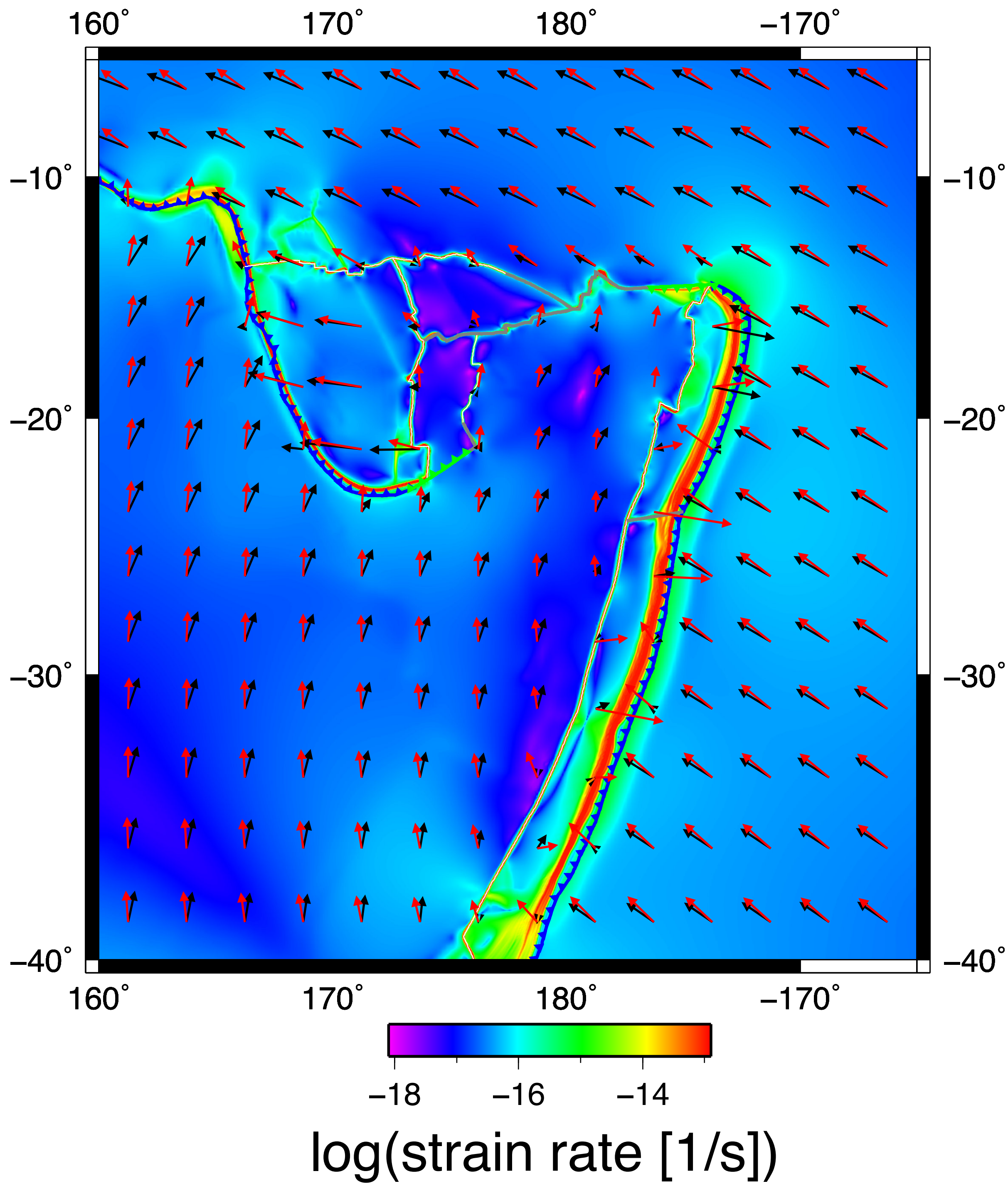
Efficient for non-linear viscosity and high-dimensional parameter space
$\eta = \eta(E, n, \sigma_y, \Gamma, .etc)$



Hu, Rudi, Gurnis & Stadler PNAS [2024]

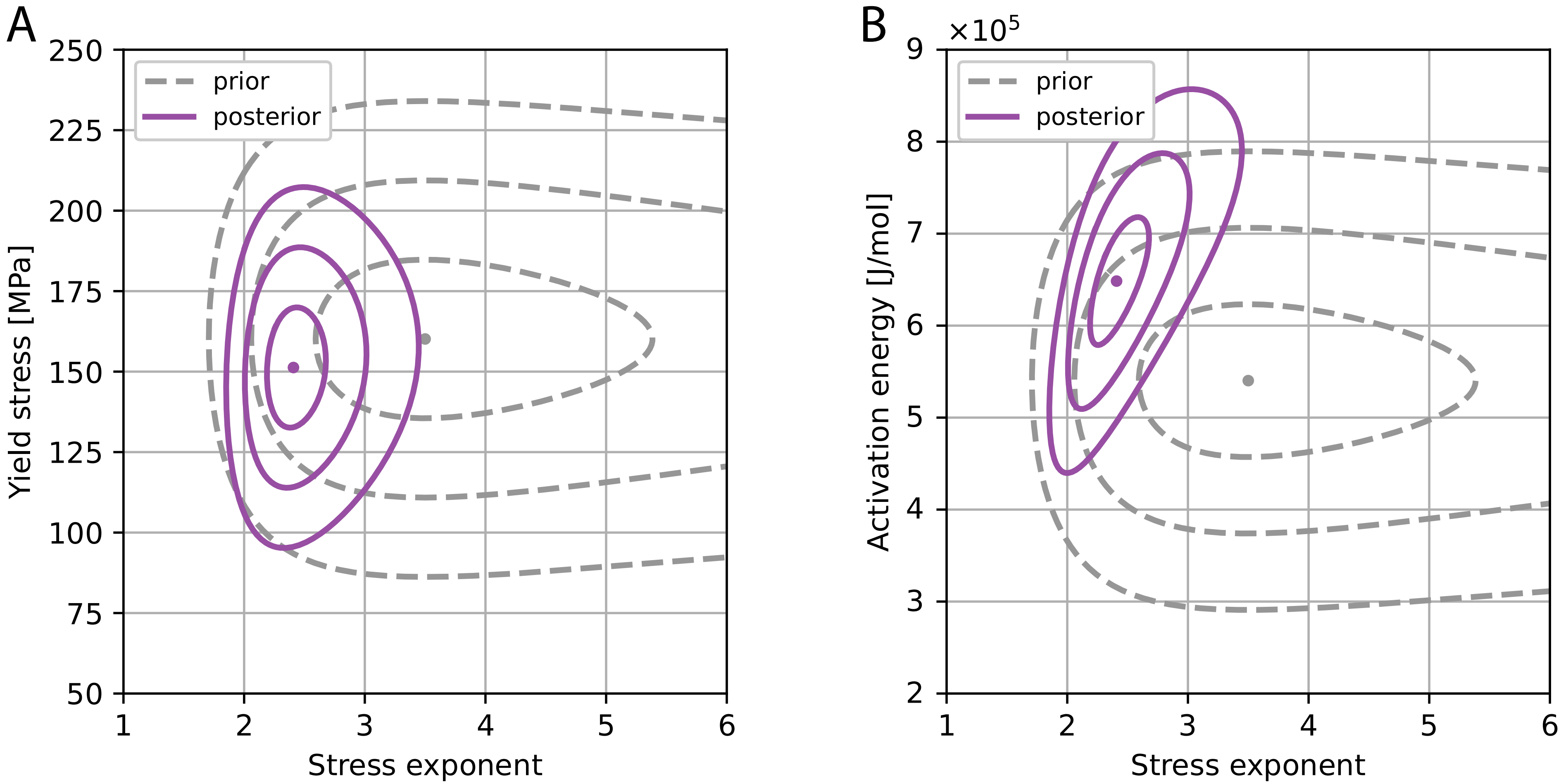
Most Coupling Factors are within a narrow range





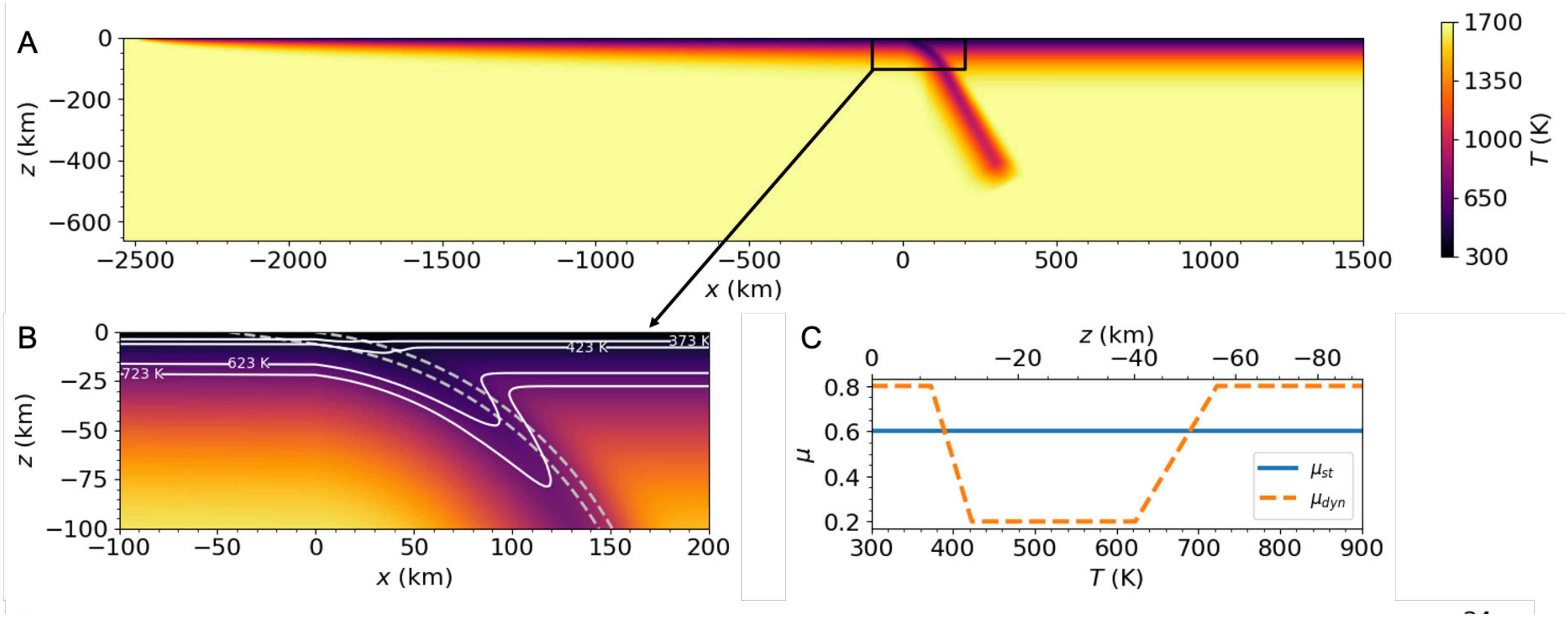
Hu, Rudi, Gurnis & Stadler [2024]

Marginals from Global Inversions



Hu, Rudi, Gurnis
& Stadler PNAS [2024]

Science 2: Replace the material in the shear zone with a Frictional Material, while the whole domain is visco-elastic



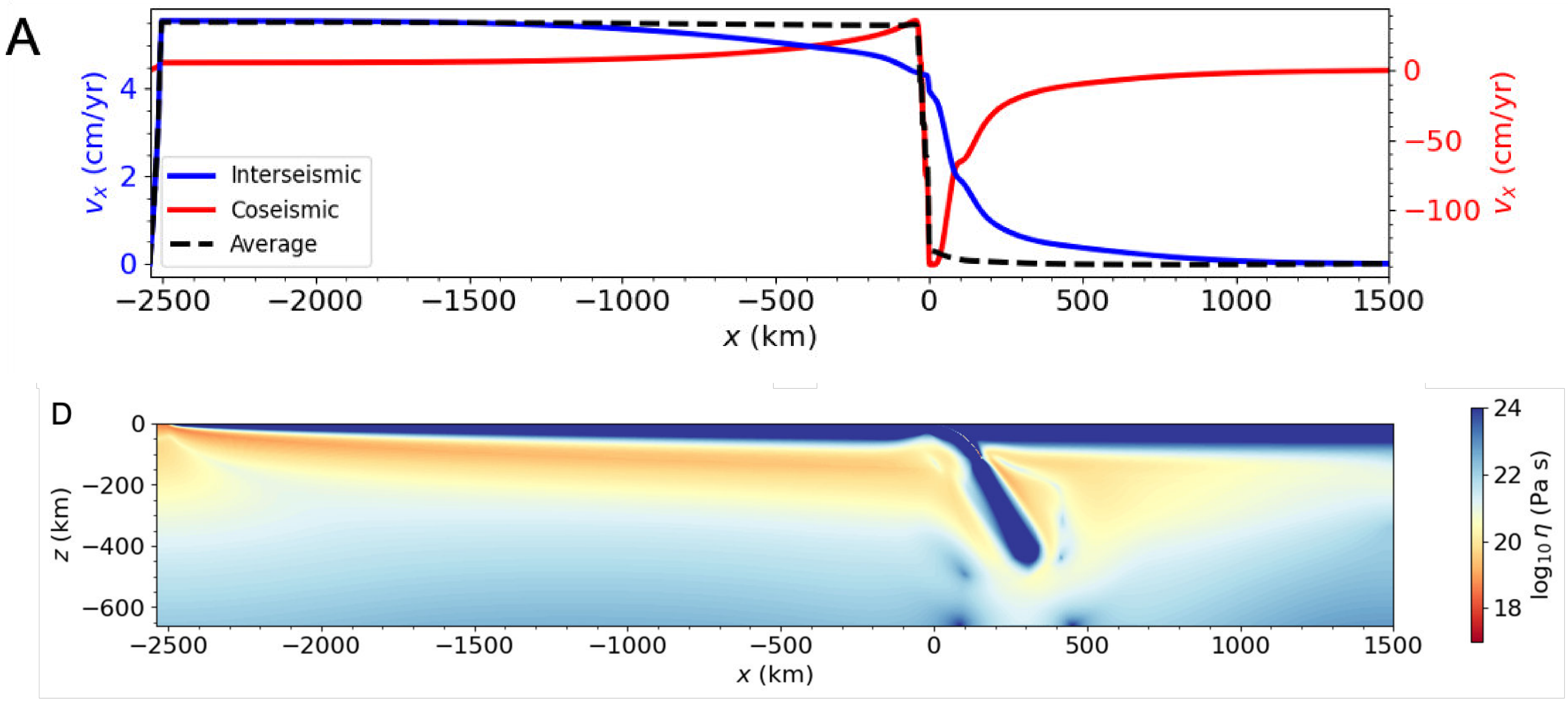
$$\tau_y = C + \mu P\ (1 - \lambda)\,,$$

$$\mu = \mu_d + \frac{\mu_s - \mu_d}{1 + \frac{V}{V_{ref}}}\,,$$

Implemented in Underworld2,
a FEM package [Moresi, et al.]

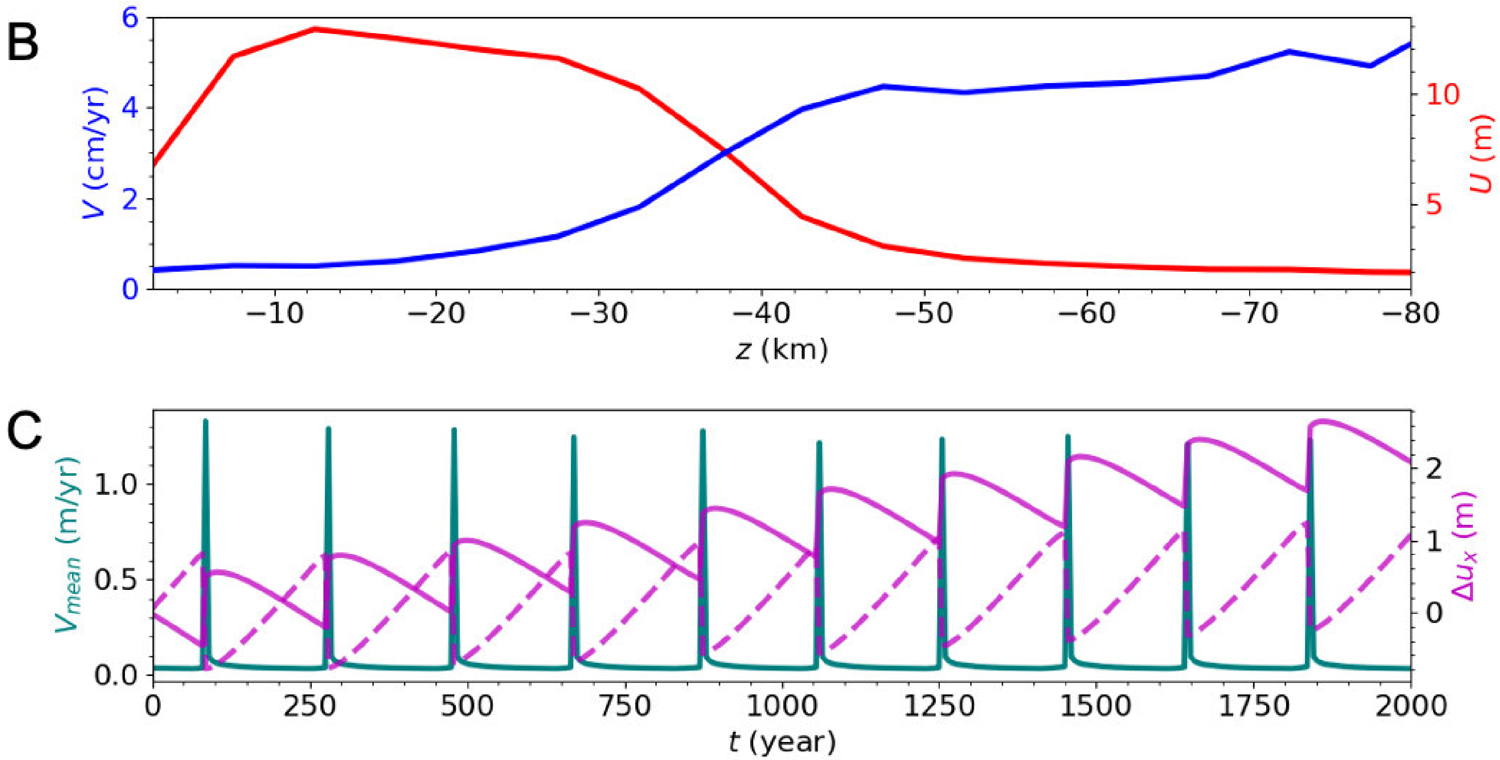
Fang, Gurnis & Lapusta, submitted [2024]

Long-term steady-state plate motion

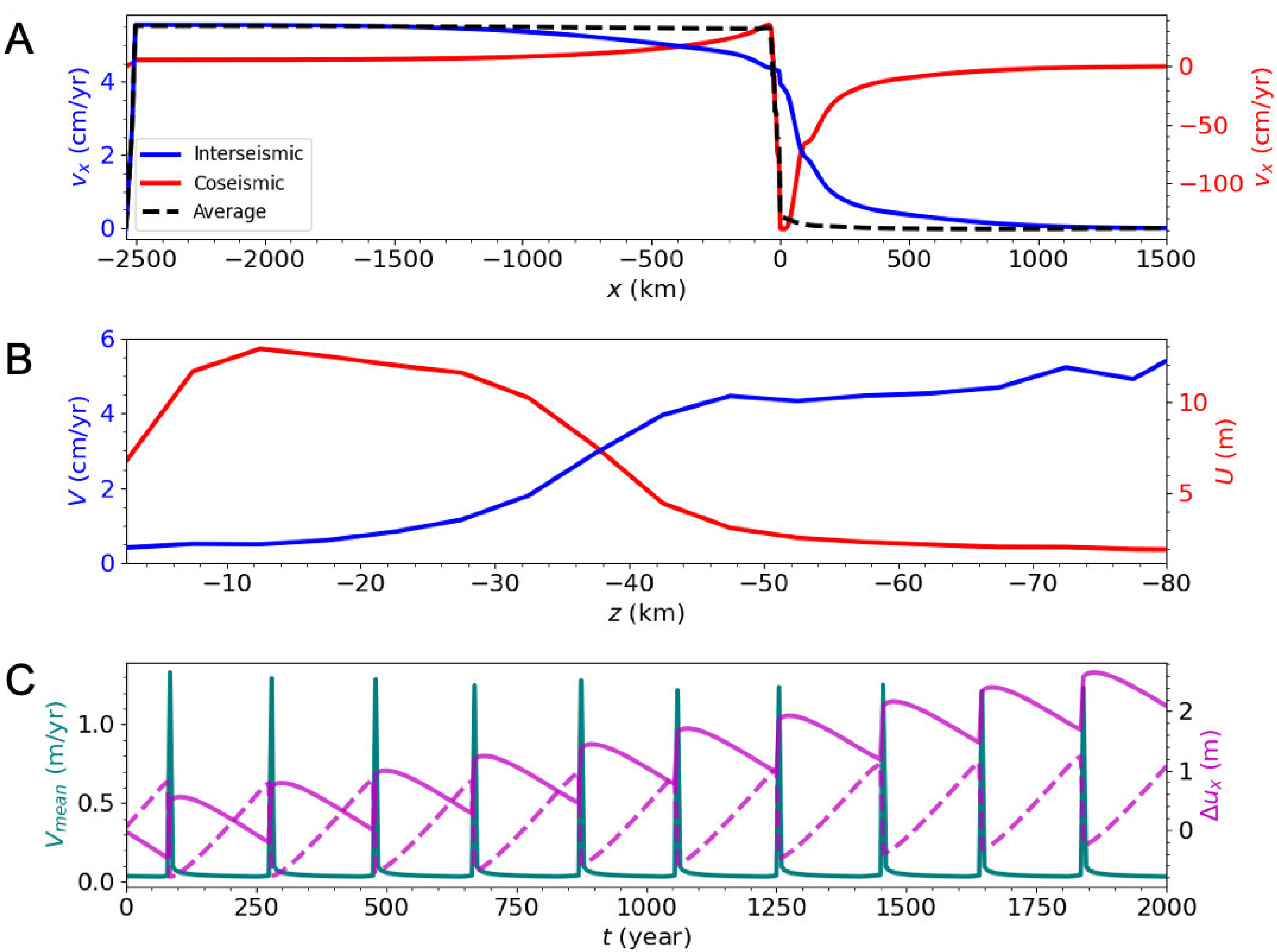


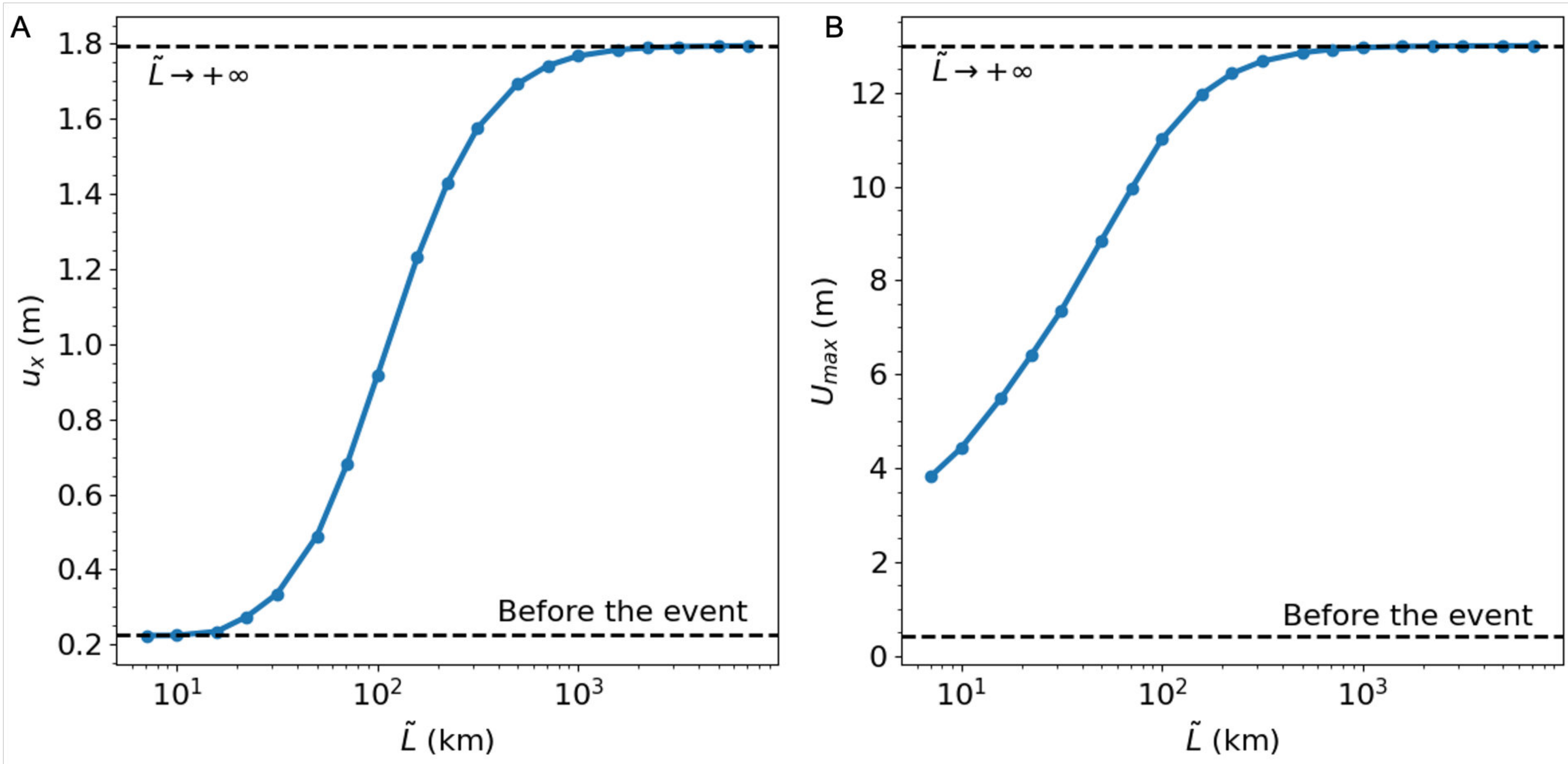
Simultaneously: Slip Events and Plate Motions

Shear Zone: Co-seismic Slip and Interseismic Velocity



Simultaneous Slip Events and Plate Motions





L=1,400 km, slip=10m and a down-dip length=50 km

Assuming elliptical slip distribution, we get

M_w ~ 9 (M_o = 2 x 10^{22} N-m) every 300 years

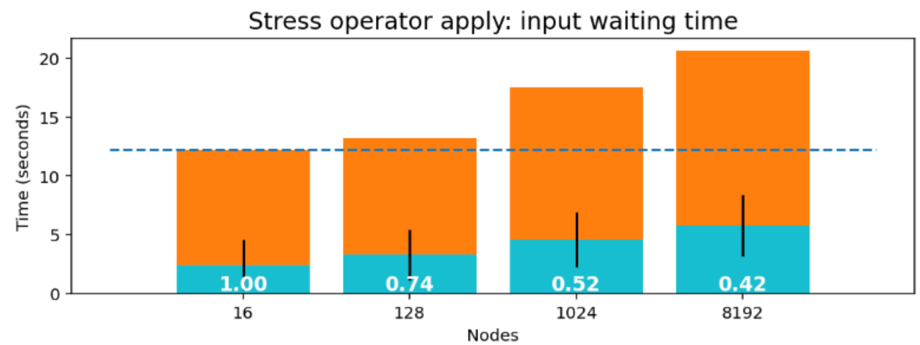
All while U_p ~ 5 cm/yr and η_a ~ 10^{19} Pa-s emerging from the dynamics

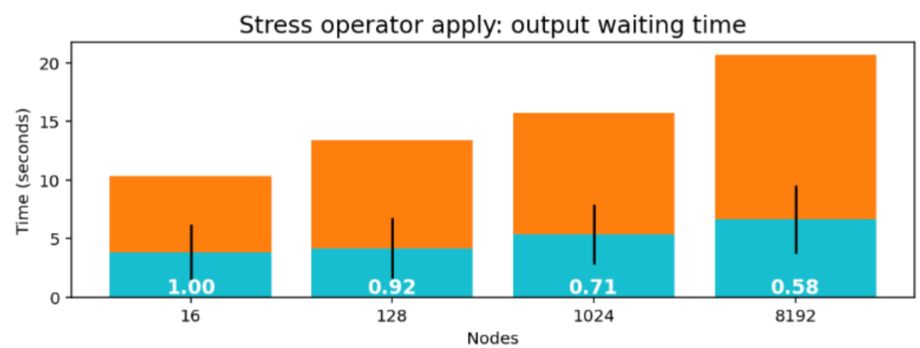
To advance from earthquake cycles to dynamic plate motions, regionally to globally, we're working to overcome computational challenges:

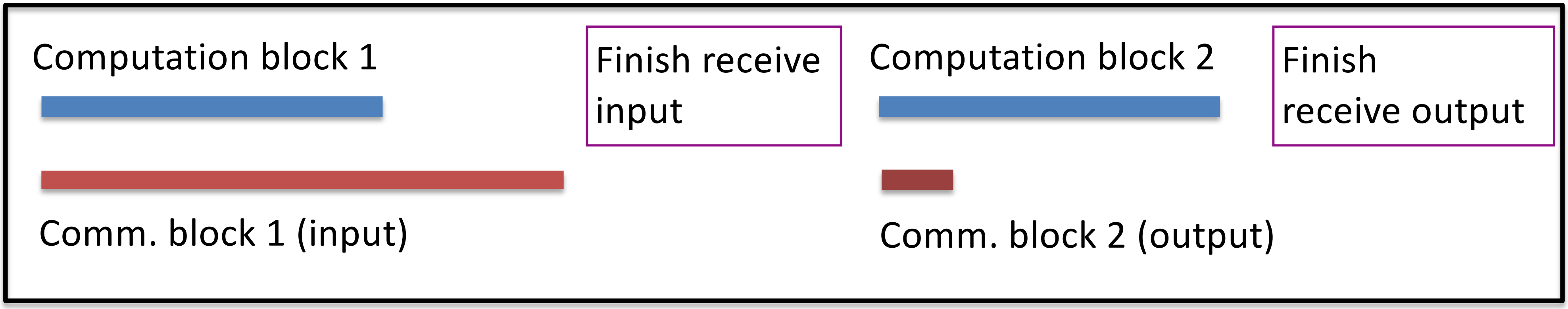
1. Increase the scaling of the Solver
2. Update materials & equations
   a. Visco-elastic system (Maxwell Model)
   b. Frictional material inside fault zones
3. achieve ~10-meter resolution inside fault zones

1. Working on better scaling: Hiding point-to-point communication during parallel matrix-vector products

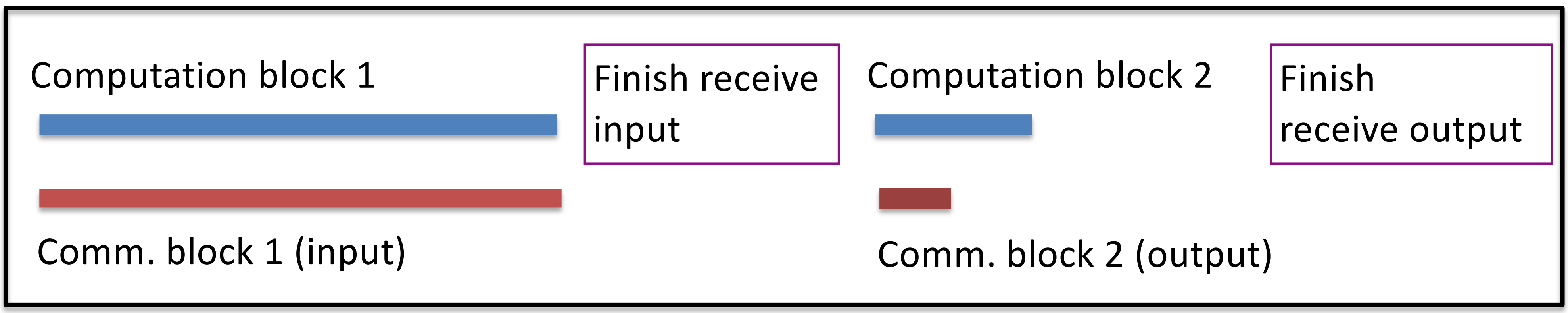
- Motivation: waiting time for input and output vectors during matvecs can make up a large portion of computation time
- Right image: weak scaling series on Frontera for one Newton step (50 GMRES iterations)
  - orange is maximum (over all processes) cumulative waiting time for input and output during matrix-vector products
  - cyan is mean across processes
- waiting time increases to 20s — almost 10% of total computation time — and this is just one of several types of matvecs
- one reason: geometry and hardware create imbalance in input/output communication time
- new method to hide communication with computation during two phases of matrix-vector product





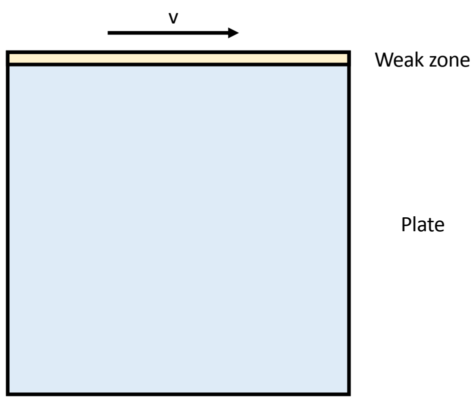


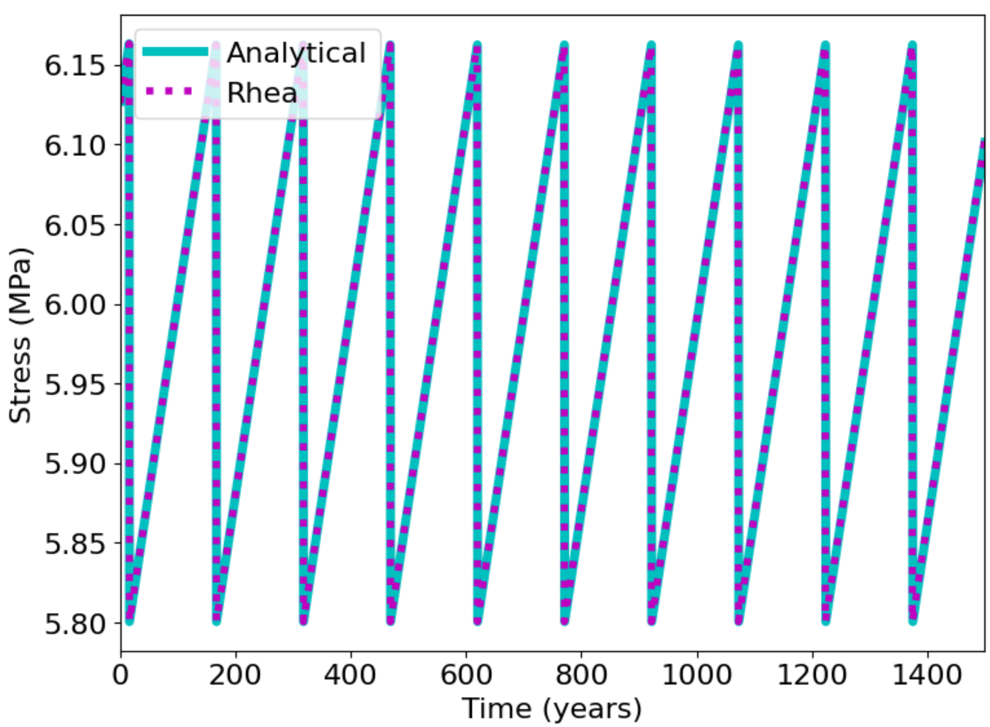
Old method: computation local to each processor split into two equally size blocks

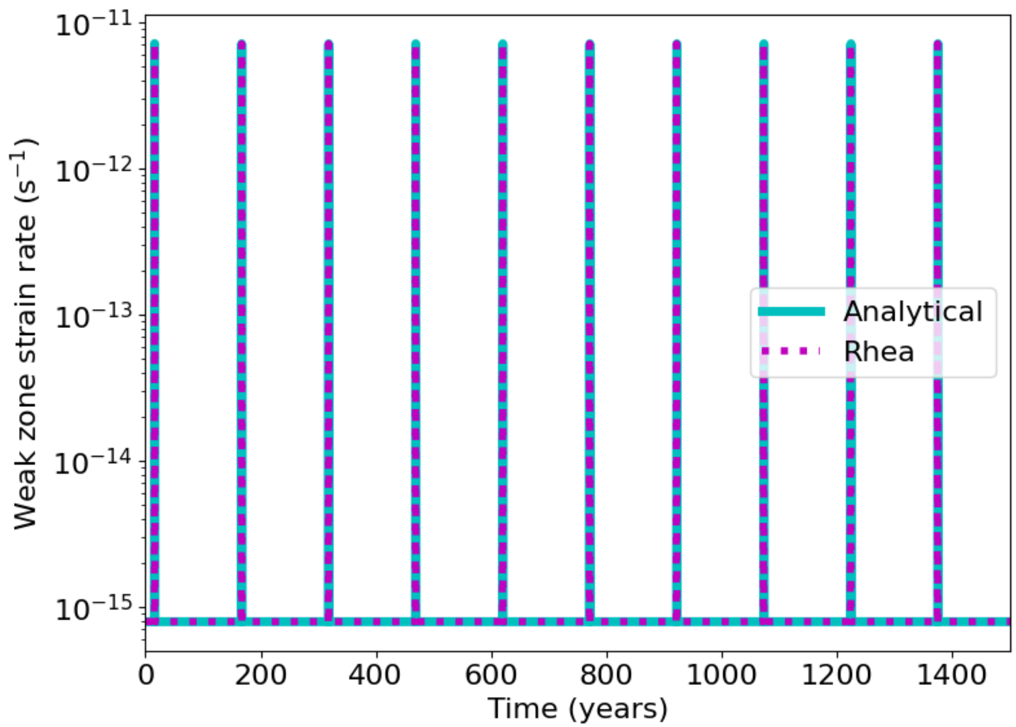


New method: computation blocks are adaptively sized to hide communication

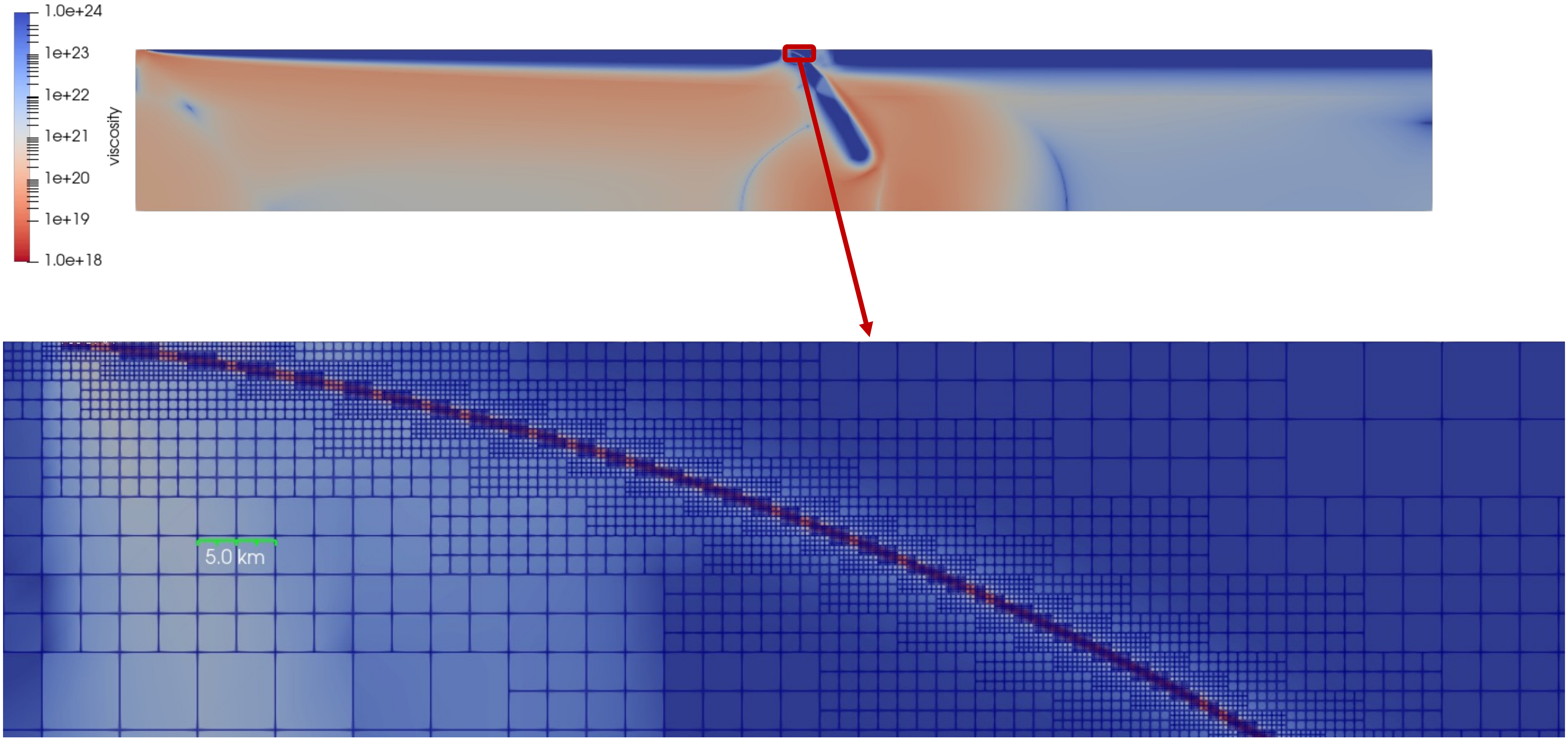
2b. Incorporation of frictional (velocity weakening) material in fault zones in Rhea with visco-elasticity







3. Currently achieving fault-zone resolutions of 75 meters (150 m elements, 2^{nd} order basis functions)



Summary

- We continue to advance Rhea, a finite element code with adaptive finite hexahedral elements with an advanced hybrid Algebraic-Geometric multi-grid Solver.
- We can solve forward & inversion problems using the Stokes equations with non-linear viscosity with yielding in a sphere
- On Frontera, we can achieve nearly ideal weak scaling on the full machine
- In global models with 1-km resolutions we demonstrated recovery of the non-linear constitutive parameters (a first)
- In visco-elastic models, we demonstrate plate tectonic to great earthquake dynamics broadly consistent with observed plate motions, mantle viscosities, and megathrust slip (a first)
- We are advancing the scalability and material models in Rhea to compute cross-time scale models at regional to global scales.